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INSTITUTE OF INFORMATION & COMMUNICATION TECHNOLOGIES

ADVANCED DIGITAL SIGNAL PROCESSING

Lab 8: Windows and Design of FIR Filters

Objective

The objective of this lab is to design FIR filters using window technique.

1 Introduction

To design linear phase FIR filters based on the windowed fourier series approach, MATLAB includes the following functions for generating the windows.

1. `W = hamming(N)` % returns the N-point symmetric Hamming window
2. `W = hanning(N)` % returns the N-point symmetric Hanning window
3. `W = kaiser(N,beta)` % returns the beta-valued N-point Kaiser window
4. `W = blackman(N)` % returns the N-point symmetric Blackman window

The above functions generate a vector `W` of window coefficients of odd length `N`. Using these functions, we can use MATLAB to design FIR filters based on the window technique.

2 Window Design Techniques

For causal Finite Impulse Response (FIR) systems, the system function has only zeros (except for poles at $z = 0$), hence known also as *all-zero* filter. Let us consider an FIR filter of length M (order $N = M - 1$)[1],

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k) = \sum_{k=0}^{M-1} h_k x(n-k) \qquad (1)$$

The basic idea behind the window design is to choose a proper ideal frequency-selective filter (which always has a noncausal, infinite-duration impulse response) and then to truncate (or window) its impulse response to obtain a linear-phase and causal FIR filter. Therefore the emphasis in this method is on selecting an appropriate *windowing* function and an appropriate *ideal* filter. We will denote an ideal frequency-selective filter by $H_d(e^{j\omega})$, which has a unity magnitude gain and linear-phase characteristics over its passband, and zero response over its stopband. An ideal lowpass filter (LPF) of bandwidth $\omega_c < \pi$ is given by,

$$H_d(e^{j\omega}) = \begin{cases} 1e^{-j\alpha\omega}, & if |\omega| \le \omega_c \\ 0, & if \omega_c < |\omega| < \pi \end{cases}$$

[1] The filter order is always equal to the number of taps minus 1

where ω_c is also called the *cutoff* frequency, and α is called the sample delay.

It can be shown easily that the impulse response is given by,

$$h_d(n) = \frac{\omega_c}{\pi} \frac{\sin \omega_c(n-\alpha)}{\omega_c(n-\alpha)}$$

Note that $h_d(n)$ is symmetric with respect to α, a fact useful for linear-phase FIR filters.

The impulse response has a sinc shape which is non-causal and infinite in duration. To obtain an FIR filter from $h_d(n)$, one has to truncate $h_d(n)$ on both sides. To obtain a causal and linear-phase FIR filter $h(n)$ of length M, we must have,

$$h(n) = \begin{cases} h_d(n), & 0 \le n \le M-1 \\ 0, & elsewhere \end{cases} \quad and \quad \alpha = \frac{M-1}{2}$$

This operation is called "windowing". In general, $h(n)$ can be thought of as being formed by the product of $h_d(n)$ and a window function $w(n)$ as follows,

$$h(n) = h_d(n)w(n)$$

Depending on how we define $w(n)$, we obtain different window designs. For example, if we apply rectangular window,

$$w(n) = \begin{cases} 1, & 0 \le n \le M-1 \\ 0, & otherwise \end{cases}$$

Use the MATLAB's built-in function `rectwin()` to draw a rectangular window of length 25.

```
>> wvtool(rectwin(25))
```

Lets say we want to design a type 1 lowpass filter using rectangular window of length $M = 45$ with cutoff frequency of $\omega_c = 0.25\pi$ radians per second. Determine the impulse response and provide a plot of the frequency response of the designed filter.

```
clc, clear all, close all
M = 45; % filter length
wc = 0.25*pi; % cutoff frequency in radians
n = 0:1:M-1; % samples between 0 to M-1

alpha = (M-1)/2; m = n - alpha;
fc = wc/pi; hd = fc*sinc(fc*m); % impulse
    response of ideal LPF
w_rec = (rectwin(M))'; % rectangular window
h = hd .* w_rec; % H(z)

[H,w] = freqz(h,1,1000,'whole'); % frequency
    response of digital filter
H = (H(1:1:501))'; w = (w(1:1:501))'; %  w =
    501 frequency samples between 0 to pi
    radians
mag = abs(H); % absolute magnitude computed
    over 0 to pi radians
```

```
db = 20*log10((mag+eps)/max(mag)); % Relative
    magnitude in dB computed over 0 to pi
    radians

% Plots
subplot(2,2,1); stem(n,h); title('Impulse
    Response'),grid
axis tight;xlabel('n'); ylabel('h(n)')
subplot(2,2,2); stem(n,w_rec);title('
    Rectangular Window'),grid
axis tight; xlabel('n'); ylabel('w(n)')
subplot(2,2,3); plot(w/pi,mag);title('Absolute
    Magnitude Response'),grid
axis tight; xlabel('frequency in \pi units');
    ylabel('Amplitude Response')
subplot(2,2,4);plot(w/pi,db);title('Relative
    Magnitude Response in dB'),grid
axis tight; xlabel('frequency in \pi units');
    ylabel('Decibels')
```

2.1 Kaiser Window

The window function is due to J. F. Kaiser and is given by,

$$w(n) = \frac{I_o\left[\beta\sqrt{1-(1-\frac{2n}{M-1})^2}\right]}{I_o[\beta]}, \qquad 0 \le n \le M-1$$

where $I_o[.]$ is the *modified zero-order Bessel* function given by,

$$I_o(x) = 1 + \sum_{k=0}^{\infty}\left[\frac{(x/2)^k}{k!}\right]^2$$

which is positive for all real values of x. The parameter β controls the minimum stopband attenuation A_s and can be chosen to yield different transition widths for near-optimum A_s. This window can provide different transition widths for the same M, which is something other fixed windows lack. In addition, the Kaiser window provides flexible transition bandwidths. Due to the complexity involved in the Bessel functions, the design equations for this window are not easy to derive. Given *passband corner frequency* ω_p, *stopband corner frequency* ω_s, *passband ripple* R_p, and *stopband attenuation* A_s, the parameter M and β are given by,

$$\text{Transition width} = \Delta\omega = \omega_s - \omega_p$$

$$\text{Filter length } M \simeq \begin{cases} \lceil\frac{A_s-7.95}{2.285\Delta\omega}+1\rceil, & A_s > 21 \\ \lceil\frac{5.79}{\Delta\omega}\rceil, & A_s \le 21 \end{cases}$$

$$\text{Parameter } \beta = \begin{cases} 0.1102(A_s-8.7), & A_s > 50 \\ 0.5842(A_s-21)^{0.4} + 0.07886(A_s-21), & 21 \le A_s \le 50 \\ 0, & A_s < 21 \end{cases}$$

2.2 Example

Lets say we want to design a digital FIR lowpass filter with the following specifications,

$$\omega_p = 0.2\pi \quad R_p = 0.25\,dB$$
$$\omega_s = 0.3\pi \quad A_s = 50\,dB$$

we choose kaiser window and determine the impulse response and provide a plot of the frequency response of the designed filter.

```
clc, clear all, close all
wp = 0.2*pi; % digital passband freq in rad
ws = 0.3*pi; % digital stopband freq in rad
Rp = 0.25; % passband ripple in dB
As = 50; % stopband attenuation in dB
tr_width = ws - wp; % transition width
M = ceil((As-7.95)/(2.285*tr_width)+1); %
    filter length
disp(['Filter length M is ',num2str(M)])
n = 0:1:M-1; % samples between 0 to M-1
beta = 0.1102*(As-8.7); % parameter beta
disp(['Parameter beta is ',num2str(beta)])
wc = (ws+wp)/2; % cutoff frequency in radians
alpha = (M-1)/2; m = n - alpha;
fc = wc/pi; hd = fc*sinc(fc*m); % impulse
    response of ideal LPF
w_kai = (kaiser(M,beta))'; h = hd .* w_kai; % H
    (z)

[H,w] = freqz(h,1,1000,'whole'); % frequency
    response of digital filter
H = (H(1:1:501))'; w = (w(1:1:501))'; %  w =
    501 frequency samples between 0 to pi
    radians
mag = abs(H); % absolute magnitude computed
    over 0 to pi radians
db = 20*log10((mag+eps)/max(mag)); % Relative
    magnitude in dB computed over 0 to pi
    radians
pha = angle(H); % Phase response in radians
    over 0 to pi radians
grd = grpdelay(h,1,w); % Group delay over 0 to
    pi radians
delta_w = 2*pi/1000;
As = -round(max(db(ws/delta_w+1:1:501))); %
disp(['Min Stopband Attenuation is ',num2str(As
    )])

% Plots
subplot(2,2,1); stem(n,hd); title('Ideal
    Impulse Response')
axis([0 M-1 -0.1 0.3]); xlabel('n'); ylabel('hd
    (n)')
subplot(2,2,2); stem(n,w_kai);title('Kaiser
    Window')
axis([0 M-1 0 1.1]); xlabel('n'); ylabel('w(n)'
    )
subplot(2,2,3); stem(n,h);title('Actual Impulse
     Response')
axis([0 M-1 -0.1 0.3]); xlabel('n'); ylabel('h(
    n)')
subplot(2,2,4);plot(w/pi,db);title('Magnitude
    Response in dB');grid
axis([0 1 -100 10]); xlabel('frequency in \pi
    units'); ylabel('Decibels')
```

3 Exercise

1. Draw following windows each of length 60.
   (a) Bartlett Window
   (b) Hann Window

(c) Hamming Window

(d) Blackman Window

Draw the magnitude response as well of each of these windows. All plots must appear on the same graph paper with your roll # on top of the graph paper.

2. Design the FIR filter for a sampling frequency of 50 kHz using window technique with following specifications,

   (a) low-pass, cut-off frequency = 5 kHz, length = 11, rectangular window

   (b) low-pass, cut-off frequency = 5 kHz, length = 21, rectangular window

   (c) low-pass, cut-off frequency = 5kHz, length = 11, Hamming window

   (d) low-pass, cut-off frequency = 5 kHz, length = 21, Hamming window

   (e) High-pass, cut-off frequency = 2 kHz, length = 21, rectangular or Hamming window

   (f) Bandpass, lower cutoff frequency = 25kHz, upper cutoff frequency = 35kHz, length = 66, Hann Window

   In each case find the filter coefficients, measure the magnitude and phase responses and comment. Plot the magnitude and phase versus real frequency in Hz

3. Type `wintool` and investigate the time domain and frequency domain representation of various window. Also investigate the effect of varying window length on the magnitude response of a window.