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ADVANCED DIGITAL SIGNAL PROCESSING

Lab 4: Convolution Sum and Correlation

Objective

The objective of this lab is to compute and plot unit impulse response, unit step response, unit ramp response of discrete time systems, convolution sum of finite sequences, autocorrelation and cross correlation of signals.

1 Convolution Sum

The convolution of two discrete time sequences $x[n]$ and $h[n]$ can be computed as follows,

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

or, equivalently,

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

This sum of products (or convolution sum) is in fact a function of n that represent the overlap between $x[n]$ and the time-reversed and shifted version of $h[n]$. The number of samples N in the output signal $y[n]$ will be $N = M_1 + M_2 - 1$. Where, M_1 is the number of samples in sequence $x[n]$ and M_2 is the number of samples in sequence $h[n]$.

In MATLAB, the function $y = conv(x, h)$ implements the convolution of two finite length sequences $x(n)$ and $h(n)$, generating the finite length sequence y. The process is illustrated in the following example.

1.1 Convolution of two impulses

Lets say we want to examine the convolution sum of two impulses, e.g. $y[n] = \delta[n] * \delta[n-1]$ using the time axis $n = [0, 1, 2]$

```
≫ x1 = [1 0 0];
≫ x2 = [0 1 0];
```

Now, we can convolve them, and create the proper time vector the output.

```
≫ y = conv(x1,x2);
≫ t = 0:length(y)-1;
```

And, to see the result graphically, let's plot the output.

```
≫ stem(t,y);
```

What can you observe about the output of this system? Try changing the order of convolution, i.e. try `conv(x2,x1)`. Does this change the output?

1.2 Convolution of an impulse and a square wave

To have better insight, let's examine the result of a convolution sum of an impulse and a square, e.g. $y[n] = \delta[n-1] * x_s[n]$. Where,

$$x_s[n] = \begin{cases} 1, & 0 \le n \le 5 \\ 0, & elsewhere \end{cases}$$

To create the signals,

```
≫ x = [0 1 0 0 0 0];
≫ xs = [1 1 1 1 1 1];
```

Now, convolve the signals and create a time vector

```
≫ y = conv(x,xs);
≫ t = 0:length(y)-1;
```

And, to plot the output.

```
≫ stem(t,y);
```

What can you observe about the output? Try shifting the time square wave and/or the impulse. How does this change the convolution output? Try to convolve two square waves $y[n] = x_s[n] * x_s[n]$ and observe the result.

1.3 Example

Determine and sketch the convolution sum of,
$x(n) = [-2\ 0\ 1\ -1\ 3]$ and $h(n) = [1\ 2\ 0\ -1]$

```
clc, clear all, close all
x = [-2 0 1 -1 3]; % x(n)
h = [1 2 0 -1]; % h(n)
y = conv(x,h); % convolution
n1 = length(h)-1;
n2 = length(x)-1;
k = -n1:1:n2;
disp(['Output Sequence y[n] = ', num2str(y)]);
figure;
subplot(311)
stem(x,'.'), title('x[n]')
subplot(312)
stem(h,'.'), title('h[n]')
subplot(313)
stem(k,y,'.'), title('y[n]')
```

To have better insight, lets do another example where, $x(n) = \{\underset{\uparrow}{1\ 2}\ 3\}$ and $h(n) = \{2\ 4\ \underset{\uparrow}{3}\ 5\}$. Using the MATLAB script.

```
clc, clear all, close all
x = [1,2,3]; % x(n)
nx = -1:1;
h = [2,4,3,5]; % h(n)
nh = -2:1;
nyb = nx(1)+nh(1); nye = nx(length(x)) + nh(
    length(h));
```

```
ny = nyb:nye; y = conv(x,h);
disp(['Output Sequence y[n] = ', num2str(y)]);
figure;
subplot(311)
stem(nx,x,'.'), title('x[n]')
subplot(312)
stem(nh,h,'.'), title('h[n]')
subplot(313)
stem(ny,y,'.'), title('y[n]')
```

2 Responses of LTI Systems

The response of a discrete time system to a unit impulse sequence is called the "unit impulse response" or simply the "impulse response". Correspondingly, the response of a discrete time system to a unit step sequence is its "unit step response". The output $y[n]$ of a causal LTI system can be simulated in MATLAB using the function "filter". In one of its forms, the function, $y = filter(p, d, x)$ processes the input data vector x using the system characterized by the coefficient vectors p and d to generate the output vector y assuming zero initial conditions. The length of y is the same as the length of x. The following example illustrates the computation of the impulse and step responses of an LTI system.

2.1 Example

Find impulse and step response of the following LTI system

$$y[n] + 0.7y[n-1]–0.45y[n-2]–0.6y[n-3] = 0.8x[n]–0.44x[n-1] + 0.36x[n-2] + 0.02x[n-3]$$

```
clc, clear all, close all
p = [0.8 -0.44 0.36 0.02]; % x(n) coefficients
d = [1 0.7 -0.45 -0.6]; % coefficients of y(n)
N = 41; % desired impulse response length
x = [1 zeros(1,N-1)]; % impulse input
y = filter(p,d,x); % impulse response (output)
k = 0:1:N-1;
stem(k,y)
title('Impulse response')
xlabel('Time index (n)')
ylabel('Amplitude')
```

To determine the step response, we replace in the above program the statement `x = [1 zeros(1,N-1)]` with the statement `x = ones(1,N)`

3 Cross Correlation Sequence

There are applications where it is necessary to compare one reference signal with one or more signals to determine the similarity between the pair and to determine additional information based on the similarity.

A measure of similarity between a pair of energy signals, $x[n]$ and $h[n]$, is given by the cross-correlation sequence $\Gamma_{xh}(k)$. Where the parameter k is called lag, indicating the time-shift between the pair of signals.

The cross correlation sequence $\Gamma_{xh}(k)$ of two discrete time signals $x[n]$ and $h[n]$ is defined as,

$$\Gamma_{xh}(k) = \sum_{n=-\infty}^{\infty} x[k]h[n-k]$$

or equivalently,

$$\Gamma_{hx}(k) = \sum_{n=-\infty}^{\infty} h[k]x[n-k]$$

Similarly, the number of samples N in the output signal will be $N = M_1 + M_2–1$.

In MATLAB, the function $conv(x, flipr(h))$ may be used to compute this sequence. Alternatively, $xcorr(x, h)$ and $xcorr2(x, h)$ may also be used.

3.1 Example

Compute and sketch the correlation sequence for the following signals.

$x(n) = [\underset{\uparrow}{0}\ 1\ {-2}\ 3\ {-4}]$ and $h(n) = [0.5\ 1\ \underset{\uparrow}{2}\ 1\ 0.5]$

```
clc, clear all, close all
t = [-5 -4 -3 -2 -1 0 1  2  3  4 5]; % x-axis
x = [ 0  0  0  0  0 0 1 -2  3 -4 0]; % input
h = [ 0  0  0 0.5 1 2 1 0.5 0  0 0]; % response
y = xcorr2(x,h);
n1 = length(x)-1;
n2 = length(h)-1;
k = -n1:1:n2;
disp(['Output Sequence y[n] = ', num2str(y)]);
figure;
subplot(311)
stem(t,x,'m.'), title('x[n]')
subplot(312)
stem(t,h,'r.'), title('h[n]')
subplot(313)
stem(k,y,'.'), title('y[n]')
axis([-10 10 -7 2])
```

A different code to achieve same outcome,

```
clc, clear all, close all
t = [-5 -4 -3 -2 -1 0 1  2  3  4 5]; % x-axis
x = [ 0  0  0  0  0 0 1 -2  3 -4 0]; % input
h = [ 0  0  0 0.5 1 2 1 0.5 0  0 0]; % response
y = [ 0  0  0  0  0 0 0  0  0  0 0];

yc = 1;
for n=min(t):max(t),
    pause(3);

    % flip h
    ht = fliplr(h);

    if n<0,
        % shift to the left
        ht = [ht(-n+1:length(h)) zeros(1,-n)];
    else
```

```
        % shift to the right
        ht = [zeros(1,n) ht(1:length(h)-n)];
    end

    y(yc) = sum(x.*ht);
    yc = yc + 1;

    subplot(2,1,1);
    stem(t,x);
    hold on;
    stem(t,ht,'filled','r');
    hold off;
    xlabel('t');
    legend('x[n]','h[n-k]',0);
    title(['n=' num2str(n)]);

    subplot(2,1,2);
    stem(t,y);
    xlabel('t');
    ylabel('y[n]');
end
```

4 Autocorrelation Sequence

Autocorrelation indicates how the signal energy (power) is distributed within the signal, and as such is used to measure the signal power.

The auto correlation sequence $\Gamma_{xx}(k)$ of discrete time signals $x(n)$ is computed as,

$$\Gamma_{xx}(k) = \sum_{n=-\infty}^{\infty} x(n)x(n-k)$$

The number of samples N in the output signal will be $N = 2 \times M - 1$. Where, M is the number of samples in the sequence $x[n]$.

The Matlab function $xcorr2(x)$ is used to find the autocorrelation function.

4.1 Example

Find the autocorrelation function of the signal,

$x(n) = [1\ 2\ 1\ 1]$

```
clc, clear all, close all
x = [1 2 1 1];
y = xcorr2(x);
n = length(x)-1;
k = -n:1:n;
disp(['Output Sequence y[n] = ', num2str(y)]);
figure;
subplot(211)
stem(x,'.'), title('x[n]')
subplot(212)
stem(k,y,'.')
title('y[n]')
xlabel('Lag index k')
ylabel('Amplitude')
```

5 Exercise

1. Compute the convolution sum for the following signals,
   (a) $x = [\underset{\uparrow}{1}\ 2\ 1\ -1]$ and $h = [1\ \underset{\uparrow}{2}\ 3\ 1]$
   (b)
   $$x(n) = \begin{cases} 1, & n = \pm 1 \\ 2, & n = 1 \\ 0 & elsewhere \end{cases} \qquad h(n) = \begin{cases} 2, & n = 0 \\ 3, & n = 1 \\ -2, & n = 2 \\ 0 & elsewhere \end{cases}$$

2. Determine and sketch the unit ramp response of the system given in Example 2.1. Hint: x=k

3. Determine the impulse response, unit step response and unit ramp response of the system described by the difference equation. $y[n] = 0.7y[n-1] - 0.1y[n-2] + 2x[n] - x[n-2]$

4. Sketch the response of the system characterized by the impulse response $h[n] = (1/2)^n u[n]$ to the input signals
   $$x(n) = \begin{cases} 1, & 0 \leq n \leq 10 \\ 0 & elsewhere \end{cases}$$

5. Compute and sketch the cross correlation for the following signals,
   (a) $x(n) = [1\ \underset{\uparrow}{2}\ 3\ 4]$ and $h(n) = [4\ 3\ \underset{\uparrow}{2}\ 1]$
   (b) $x(n) = [\underset{\uparrow}{4}\ 2\ 1\ 5]$ and $h(n) = [\underset{\uparrow}{3}\ 5\ 2]$

6. Compute and sketch the autocorrelation sequence for the following signals,
   (a) $x(n) = [2\ 3\ 5\ 6]$
   (b) $m(n) = [2\ 1\ 1\ 1\ 2]$